OUNDLE

School

2020 Academic Scholarship

Mathematics

Paper II

Time Allowed: **2 hours**

Calculators may NOT be used for this paper

Instructions to candidates:

- You are not expected to have time to do all the questions.
- Answer on the lined paper provided
- You may answer the questions in any order.
- Choose those questions which you think you can answer best.
- **Remember to show your working and clearly show the method you are using**.
- Answers should be given to 3 significant figures where appropriate.
- π may be taken as 3.14.
- **The number of marks for each question is show in square brackets**

1. The correct solution to the calculation below is answer A: 135135. Clearly explaining your reasoning for each, without doing the whole calculation, how can you know that the other answers are incorrect?

   1 x 3 x 5 x 7 x 9 x 11 x 13 ?

   A: 135135 B: 142061 C: 106128 D: 14576 E: 12765

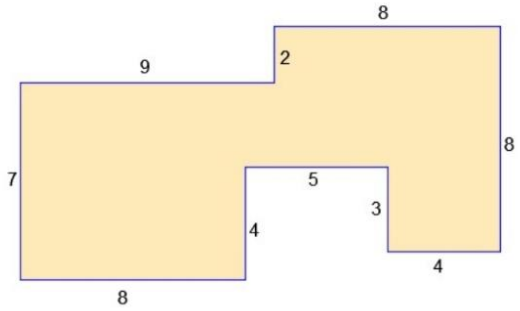
   [3]

2. Starting from the number 1, your goal is to get to the number 17 using only these actions:
   - Add 1
   - Multiply by 3

   What is the minimum number of actions it takes to get to 17? [2]

3. Kenny's wife Kerry is an architect and she is designing a new house for them to live in. She shows Kevin the floor plan shown in the diagram, with lengths in metres, and with all right angles.

   Find the area of the proposed floor of the house.

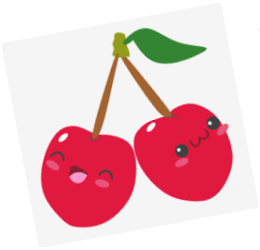


   [3]

4. Three brothers stayed in a house with their mother. One day, their mother brought home some cherries.

   Alex woke up first. As he was hungry, he ate $\frac{1}{2}$ of the cherries plus one extra cherry and headed out.

   Brian woke up next. As he was hungry, he takes $\frac{1}{3}$ of the remaining cherries – and after he ate, he put two cherry back in the basket.



Charles woke up next. As he was hungry, he ate $\frac{5}{6}$ of the remaining plus one extra cherry and headed out.

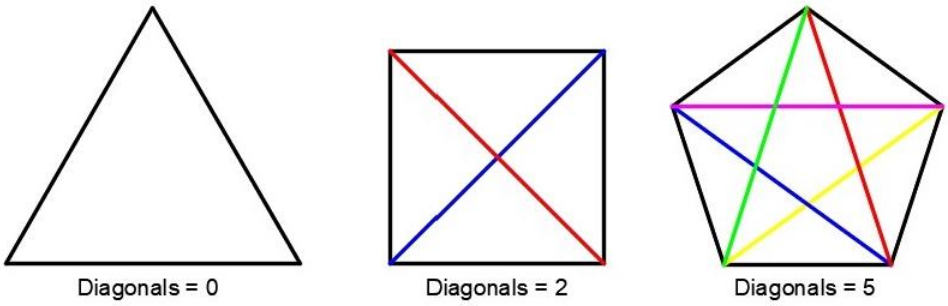
Their mother came home and saw seven cherries in the basket. How many cherries were there initially?

[4]

5. An equilateral triangle has no diagonal, a square has 2 diagonals, and a regular pentagon has 5 diagonals.

   How many diagonals does a regular 15-sided polygon have?

   Can you find a formula for the number of diagonals of an n-sided polygon?



[4]

6. How many 2-digit prime numbers remain prime when the order of their digits is reversed?

[4]

7. Larry, Colin and Mike deliver public health notices to houses in a very long road but in a peculiar way. Starting at the end of the road, Larry delivers a leaflet to house #12 and then every twelfth house thereafter. In a similar way, Colin delivers to house #9 and every 9^{th} house thereafter while Mike delivers to house #15 and every 15^{th} house thereafter.

   a) What are the first 3 house numbers that they will all deliver leaflets to?
   b) How many of the first 1000 houses in the street, will receive exactly 2 leaflets?

[6]

8. You, a newbie cop, just gathered intel about four thieves, Andrew, Orville, Nottica and Xorax have run off from the bank with a bunch of coins. You chase them to a dead end. Seeing no way out, the four decide to play a mind game with you.

Andrew: I only have coins if every one of them have coins too.

Orville: If at least two of them don't have coins, I won't have any.

Nottica: I only have coins if Andrew has some too.

Xorax: I only have coins if Orville and Nottica both either have coins or none at all.

From your intel, you know that all of them have a syndrome that prevents them from telling lies. With your skill, you could not apprehend all of them, so you have to make sure you get some of the coins back.

Who is guaranteed to have coins? (justify your answer)

[3]

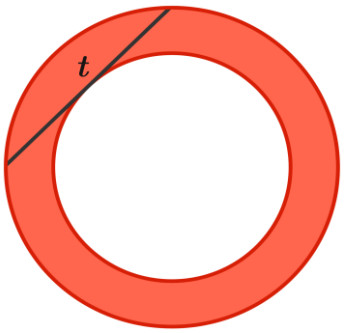
9. The sum of the ages of my five nieces is 47. Their ages are positive integers, and any two of them have a common factor (other than 1).

How old is the eldest?

[4]

10. This annulus has been formed from two concentric circles. A chord of length t of the larger circle is then drawn which just touches the inner circle.

If t = 10cm, what is the the area of the annulus (coloured red)?



[4]

11. A mysterious stranger approaches you and hands you 1023 lemons and 10 bags. The stranger says, "Arrange these lemons in these bags however you like, but soon you will be required to give a specific number of lemons (by handing over some bag or bags), and you cannot transfer the lemons between bags once you have arranged them all. Good luck!" Then, she walks away.

After you finish arranging the lemons, a friend suddenly approaches you and frantically asks, "Quick! There's no time to explain! I need exactly n lemons!"

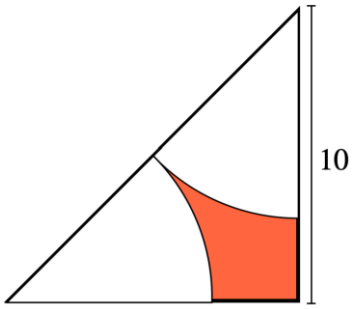
Can you fulfil your friend's request for any positive integer n less than or equal to 1023? (Justify your answer)

[3]

12. If I write 5! this means 5 x 4 x 3 x 2 x 1 which is 120. If I calculated the value of 30!, how many zeroes would it end with?

[6]

13. Two identical sectors of a circle are inscribed in an isosceles right triangle with 2 sides of length 10 units. What is the area of the red region to the ***nearest integer***?



[4]

14. The number 63 can be written as the sum of 2 consecutive integers as 31 + 32 = 63.

    In how many ways can 63 be written as the sum of a sequence of more than two consecutive integers, where all numbers in the sequence are less than 63?

[6]

15. Two communication satellites are in Earth's orbit (on the same axis) at altitudes of 96 miles and 137 miles, respectively.

If a signal must be sent between them in a straight line without Earth obstructing it, what is the **maximum distance** (to the nearest hundred miles) that the satellites can be from each other? (some reasonable rounding in your working is appropriate)

[Assume that Earth is a perfect sphere and that its radius is 4000 miles]

[6]

16. Is there an integer n>1 such that n! is a square number? (justify your answer)

[3]